

# Simple Linear Regression

## 11.4 Hypothesis Tests about the Slope $\beta_1$

1. Researchers looked at the link between the popularity of elementary school aged children (8 – 11 years old) and their ability to recognize emotion in facial expressions. The 11 participant students were each assessed by their peers for popularity in an initial phase of the experiment. The mean popularity score given to the students by their peers is reported below in the first row. The score was based on a five-point scale with five indicating they are well liked and a score of one indicating they are disliked. In the second row, the number of correct answers each student had on an emotion-recognition test is recorded. The test involved inferring the emotional state of people in photographs. Use a 2% level of significance to test the claim that a linear relationship exists between popularity and scores on the emotional recognition exam.

(Note:  $SS_{xx} = 10.1964$ ,  $SS_{yy} = 546.1818$ ,  $SS_{xy} = 72.3545$ )

Subject	1	2	3	4	5	6	7	8	9	10	11
Popularity	1.3	2.4	1.8	4.3	3.1	3.9	3.2	2.1	4.1	1.9	2.5
Emotional Recognition	10	16	14	32	25	25	26	16	29	12	18

2. Psychologists wanted to know if the order of questions in a survey of happiness could have an effect on the correlation between responses. They ran two studies on college students where they asked about happiness and recent dating history. In the first study, they first asked ten single students to rate their happiness on a visual-analog scale, and then they asked them how many dates they had gone on in the last six months. Using the results and a 5% significance level, test the claim that in this order happiness and number of dates are positively related. (Note:  $SS_{xx} = 99.6$ ,  $SS_{yy} = 19.06$ ,  $SS_{xy} = -3.9$ )

Subject	1	2	3	4	5	6	7	8	9	10
Number of dates	10	5	6	2	1	0	8	3	2	1
Happiness	5.1	4.2	7.2	7.9	4.6	5.2	3.8	3.4	4.1	5.5

3. The psychologists from problem 2 surveyed another ten students and asked them the same questions as in question 2, except they switched the order of the questions. This meant they asked about the number of dates before asking the students to assess their happiness. Use the results below and a 5% significance level to test the claim that in this

order happiness and number of dates are positively related. (Note:

$$SS_{xx} = 93.6, SS_{yy} = 14.945, SS_{xy} = 35.6)$$

Subject	1	2	3	4	5	6	7	8	9	10
Number of dates	9	7	5	2	3	1	9	2	0	4
Happiness	7.1	7.2	6.2	4.2	5.4	4.3	6.9	4.4	3.9	5.9

Answers:

1. The preliminary calculations and final answer:

$$SS_{xx} = 10.1964, SS_{yy} = 546.1818, SS_{xy} = 72.3545$$

$$\hat{\beta}_1 = 7.0961$$

$$SSE = 32.7483$$

$$S^2 = 3.6387$$

$$S_{\beta_1} = 0.59736$$

Claim: a linear relationship exists between popularity and scores on the emotional recognition exam.

$$H_0 : \beta_1 = 0$$

$$H_a : \beta_1 \neq 0$$

$$\text{Test Stat} : 11.879$$

$$\text{Critical Values} : \pm 2.821$$

Reject the null, support the alternative

The sample data supports the claim. This means that a child who recognizes emotional states in others easily tends to be more popular with his/her peers.

2. The preliminary calculations and final answer:

$$SS_{xx} = 99.6, SS_{yy} = 19.06, SS_{xy} = -3.9$$

$$\hat{\beta}_1 = -0.03916$$

$$SSE = 18.9073$$

$$S^2 = 2.3634$$

$$S_{\beta_1} = 0.1540$$

Claim: happiness and number of dates are positively related.

$$H_0 : \beta_1 \leq 0$$

$$H_a : \beta_1 > 0$$

$$\text{Test Stat} : -0.254$$

$$\text{Critical Value} : 1.860$$

Do not reject the null, do not support the alternative

The sample data does not support the claim. This means that there does not seem to be a correlation between number of dates and happiness.

3. The preliminary calculations and final answer:

$$SS_{xx} = 93.6, SS_{yy} = 14.945, SS_{xy} = 35.6$$

$$\hat{\beta}_1 = 0.3803$$

$$SSE = 1.4048$$

$$S^2 = 0.1756$$

$$S_{\beta_1} = 0.04331$$

Claim: happiness and number of dates are positively related.

$$H_0 : \beta_1 \leq 0$$

$$H_a : \beta_1 > 0$$

$$\text{Test Stat} : 8.780$$

$$\text{Critical Value} : 1.860$$

Reject the null, support the alternative

The sample data supports the claim. This means that there seems to be a positive linear relation between number of dates and happiness. This is different from what we saw in problem 2, but in this study, they asked the questions in a different order. By putting the number-of-dates question first, the researchers subconsciously primed the participants to associate their dating life with happiness. So collectively, the two experiments show that the order of the questions in a survey could affect the outcome.